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THE MATHEMATICS TEACHER

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GRAMMAR SCHOOL MATHEMATICS.

BY HARRISON E. WEBB.

The opinion is freely expressed nowadays that the present is a transition period in American education. People who hold this view are not agreed as to just what is the nature of the transition. Their judgments on this point are as varied as are their educational creeds. One prominent educator, finding that his favorite psychological hobby will not bear him up the steep and rugged path of mathematics, declares that mathematics ought to be abolished, as far as the schools are concerned. This, to paraphrase Mr. Chesterton, is like trying to get out of having to skin a cat by denying the cat. A world without mathematics is, to say the least, a difficult thing to imagine.

It is likely that the contemporary educational literature of any decade will be found to refer to that period as one of transition. But of the ten years past we may at least say this: that the border lines between college and university mathematics, and between high school and college mathematics have been gradually broken down; and that under the inspiration of many admirable books written to enliven and improve the teaching of mathematics, and of excellent periodicals devoted to the same purpose, and of societies of mathematics teachers which include in their membership representatives of both high schools and colleges, teachers of all grades of mathematical work have come to appreciate the essential unity of their science and the supreme importance of its mission, from its simplest elements to its most advanced development.

Probably the most important of the results of this improved acquaintance has been, on the one hand, the disposition of high school teachers to extend the bounds of their mathematical knowledge into more advanced fields of study, and on the other, the willingness of college and university instructors to devote much of their time and attention to the consideration of the difficulties which high school teachers have constantly to encounter.

Meanwhile, so far as the writer is able to ascertain, the teacher of mathematics in the grammar school has been left to himself. This is due probably to several conditions.

In the first place the fact is frequently overlooked that there is such a thing as grammar school mathematics. The science itself has been thought to begin with algebra, in the high school. But those whose early mathematical education extends twenty or more years back will have no difficulty in recalling that there was more mathematics, and better mathematics, and harder mathematics in the last of their arithmetic than in the first of their algebra. If this is not true to-day, may it not be due to the fact that certain educational theorists, in the interests of some fantastic lesson-plan, have squeezed most of the mathematics out of arithmetic, leaving what Professor D. E. Smith has well characterized as a mess of historical and economic facts without any vital unity?

In the second place, the conditions under which mathematics is taught in the grammar school make it all but impossible that there should be any vitality in the work. In many, if not most, of American grammar schools, teachers in the seventh and eighth grades are presumed to be equipped to teach mathematics, English, geography, history, elementary science, civics, elocution, and often a half dozen other subjects, and this in spite of the fact that a large majority of these teachers have been denied the privilege of a college education. If it is asked how in the name of great Euclid teachers can be found with sufficient intelligence to give adequate instruction in so many branches, the answer is likely to be given, though not stated quite so baldly, that intelligence is not what is asked of them, as a rule, by those in authority, but obedience; and that the superior intelligence which inheres in their superiors in administration is able to tell them what to do. The answer *might* be given, however, that in spite of the enormous burden of daily routine, and in spite of the blind adherence of many educational administrators to outworn "principles," many grammar school teachers have by dint of years of hard work so mastered the principles of their mathematics that they have given examples of good teaching which high school and college teachers would do well to emulate.

The content of grammar school mathematics has, however, been left largely to chance, or, what is worse, to psychology. It is fitting, therefore that this field should be thoroughly investigated with reference to the soundness of the fundamental principles which are there presented, and to the appropriateness of their lines of application. It requires, moreover, no lengthy argument to show that the success of high school teaching is inseparably bound up with the right sort of grammar school work in the field of mathematics.

It was with this thought in mind that the writer offered at last winter's meeting of the Middle States Association a resolution looking toward the opening of this question for discussion, his temerity being adequately rebuked by his appointment as chairman of a committee to consider the grammar school situation.

As is usual with committees, its work has progressed slowly owing to more pressing demands upon the time of its membership. But it is safe to anticipate its action in one or two particulars, in the interest of economy of time.

It is desirable that the Association welcome to its membership grammar school teachers and principals who are interested in mathematics; and to the end that the proceedings of the Association be of interest to grammer school teachers, it is to be hoped that questions which relate particularly to their field of work be brought before the Association, and that the grammar school point of view be presented as forcefully as may be. It would be as impractical and as discourteous as well, for high school teachers to attempt to dictate to the grammar schools in this business, as it seemed not so very long ago when the colleges felt free to dictate to the high schools the character of their mathematical work.

It is believed, moreover, that recent investigations into the fundamental principles of mathematics, and the bringing to American teachers of an idea of remarkable work in this field which has for some time past been in progress in the European countries may be found to have an important bearing upon elementary method and content, in the field of grammar school work, as it has already been found to have in the case of high school mathematics.

The problem of the grammar school teacher is to a great extent one of administration. Into this we cannot enter in detail, but one may safely urge that wherever possible, something be done to enable the grammar school teacher to narrow the field of his instruction.

In line with this desired change, a plan has recently been urged by men of experience in school administration which involves the reorganization of the school systems of the country on the basis of a new division of the school period into six years of grammar school and six years of high school. This plan has met with widespread approval as a theory, and has been pursued successfully in some localities. The various departmental associations, such as our own, are not likely to be called upon to pass upon this plan. But we should be wide-awake as to its possibilities, and have in mind some definite recommendations as to the character of a six year course.

This plan leaves the earlier years of mathematical teaching about as they are now, presumably. They in time will demand consideration also. But as the seventh and eighth years are at present the center of interest, the committee has addressed itself to the task of ascertaining what is the present content of the mathematics of those years, and at the same time, what work would be suitable for those years under the "six-and-six" plan.

The question as to whether algebra should be taught at all in the seventh or eighth years is apparently still an open one. In some schools a certain amount of algebra has been made a part of the course of study, only to be discarded later. It is the opinion of the committee that this question should not be decided on general principles, without attention to detail, but that the whole situation should be subjected to a searching analysis.

Among the questions which arise is the one as to what constitutes algebra in this connection. It is to be noted that the arithmetical operations upon letters may be taught without reference to the solution of equations; as for example, in the formulation of rules and principles. Also that negative number may be considered apart from literal forms and from equations. Even the axioms may be applied to the solution of literal problems in equation form without any considerable knowledge of solution of equations in the abstract. In the field of arithmetic proper, an examination of the problems shows a surprisingly large number of topics which (not all in one school, usually) are capable of inclusion. It appears also that there are many diverse points of emphasis to be considered. A glance at the whole gives an impression that proper instruction in arithmetic alone in the grammar school constitutes in itself a very difficult problem.

The writer is strongly of the opinion that familiarity with literal quantities, with the laws of exponents, and with the operations upon negative number, as well as the solution of simple problems by means of equations may well be included in the seventh and eighth years of the grammar school, together with the inversion of simple formulae by algebraic methods; while the ninth or first high school year of mathematics should be devoted in part at least to certain portions of the arithmetic.

He also feels that there is need for much greater emphasis upon the *principles* of arithmetic and algebra, and their definite formulation.

The question of grammar school geometry is also of interest. A striking example of the loose thinking of eminent educators who are not teachers of mathematics is to be found in the oft-repeated statement that geometry should precede algebra in point of time, because it is "more concrete." Here again there is a question of what is meant by geometry. If geometry is taken to mean what it never meant to its founders, viz., measurement of lines in more directions than one, with measurement of areas and volumes, that sort of "geometry" may be easier than some parts of algebra. But if by geometry is meant the study of the principles of bidimensional and tridimensional magnitudes in their essential relations, it is hard to see wherein it can possibly be simpler than an elementary science which is limited to linear magnitude. As a matter of fact, the testimony of hundreds of students is to the effect that "algebra was easy, but this geometry is terrible." Even from the point of view of interest, wherein, it might be asked, is a triangle more interesting than a radical sign?

Finally, and as of the greatest importance, arises the question as to the way in which high school teaching should relate itself to what has been given previously in the grammar school. It would appear that a careful study of the grammar school situation is likely to influence high school teaching very profoundly.

Central High School,
Newark, N. J.

[The committee has prepared a questionnaire, to be accompanied by the following letter, which sufficiently explains its purpose.]

It has now come to be widely recognized that the transfer of a pupil from one school to another of more advanced grade and of somewhat different aims and purposes usually entails some loss of his time and waste of his energy.

Convenience, however, has decreed that the school training of a boy or girl between the (average) ages of six and eighteen shall be divided into two parts. It is important that the work of these two parts should be related as closely as possible. Neither can well be regarded as more important than the other; nor is either in a position to prescribe the limits which shall be set for the other.

In America it is an all but universal custom that the first eight years of the training of a boy or girl shall be spent in primary and grammar school, and the last four years in high school. It is now proposed in many quarters that the change from one type of school to the other shall be made two years earlier (the six-and-six plan).

It is fitting that the Association of Mathematics Teachers of the Middle States and Maryland should devote some time and attention (*a*) to the mathematical work of the seventh and eighth years generally, and (*b*) to possible mathematical courses under the proposed plan. For the purpose of laying these matters before the association a committee has been appointed by the president of the association.

In order to collect data on which to proceed the committee has prepared a questionnaire. It is proposed to submit the questions to

(*a*) High school principals and teachers, to ascertain their opinions as to what work should precede the usual high school course in algebra.

(*b*) Grammar school principals and teachers, to ascertain the proper scope of the work of these two years, in their opinion.

(*c*) Administrators, who have encountered this problem and acted accordingly.

(*d*) Heads of normal schools and colleges, who are responsible for the preparation of teachers.

It is desired that each person should submit two sets of answers, the first relating to the present "eight-and-four" plan, and the second to the proposed "six-and-six" plan.

It is the firm conviction of the committee that the success of either plan is largely in the hands of the teachers, and that general exchange of opinion is highly important at this time.

Your co-operation is earnestly requested, in the form of an early reply.

Yours very truly,
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HOWARD F. HART,
Montclair High School, Montclair, N. J.,
AGNES LONG,
William Penn High School, Philadelphia, Pa.,
JOHN H. MINNICK,
Horace Mann School, New York, N. Y.,
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H. E. WEBB, *Chairman,*
Central High School, Newark, N. J.

Questionnaire.

(*a*) The questions given below are intended to refer specifically to the teaching of mathematics in the seventh and eighth school years, but not to preclude consideration of the same topics in years below the seventh.

(*b*) The term "mathematics" is extended to include practical applications to commercial and scientific problems of common experience, without detailed study of the underlying theory of such problems.

(*c*) For convenience the topics and principles which are usually included under "beginning algebra" are divided into two groups; the first called *literal forms,* and including applications of arithmetical principles to literal numbers in combination; and the second called *equations,* and including the solution of problems by the use of one or more unknown quantities. Arithmetical problems solved by equations should be included in this last named group.

(*d*) It is desired that at least two copies of each question paper be returned, the first to relate to the seventh and eighth grades of grammar school, and the second to the first two years in high school under the "six-and-six" plan. Other copies will be furnished and welcomed in return, if it is desired to differentiate between courses with various objects in view.

(*e*) Questions may be answered briefly by the words "Yes" or "No"; or, in the case of topics, by writing the characters 7, 8, 7–8, or × in the proper spaces to signify that topics should be taken in the seventh grade, the eighth grade, both grades, or not at all; but more extended expression of opinion will be welcomed by the committee.

1. Should the entire matter of grammar school arithmetic be covered either in review or as new work (*a*) in the seventh grade?...........(*b*) in the eighth grade?...........(*c*) in the two grades together?.............

2. On a basis of ten points as a maximum, zero as a minimum, indicate the relative importance of the following aspects of arithmetical teaching in the seventh and eighth years:

	7th.	8th.	
(1) Clear comprehension of essential principles	....	....	....
(2) Ability to apply principles to unfamiliar cases	....	....	....

(3) Accuracy in calculation	....	....	....
(4) Rapidity in calculation	....	....	....
(5) Mastery of short methods in calculation	....	....	....
(6) Neatness in appearance of work	....	....	....
(7) Ability to perform operations mentally	....	....	....
(8) Literal memory of rules	....	....	....
(9) Analytic insight into problems	....	....	....
(10) Ability to write solutions in symbolically	....	....	....
(11) Ability to state solutions in general terms	....	....	....
(12) Practical measurement	....	....	....
(13) Estimating of results	....	....	....
(14) Checking of results to insure accuracy	....	....	....

(Please add any others.)

3. Should any rules be taught *verbatim?* If so, which of the following? Rules for

	7th.	8th.	
(1) Addition and subtraction of decimals	....	....	....
(2) Multiplication of decimals	....	....	....
(3) Division of decimals	....	....	....
(4) Reduction of common fractions	....	....	....
(5) Addition and subtraction of common fractions	....	....	....
(6) Multiplication of common fractions	....	....	....
(7) Division of common fractions	....	....	....
(8) Reduction of fraction to mixed number	....	....	....
(9) Reduction of mixed number to a fraction	....	....	....
(10) Tests for exact divisors..2..3..4..5..6..8..9..10	....	....	....
(11) General principles of division (any)	....	....	....
(12) Highest common factor	....	....	....
(13) Lowest common multiple	....	....	....
(14) Euclidean method of H. C. F. and L. C. M.	....	....	....
(15) Reduction of denominate numbers	....	....	....
(16) Operations on denominate numbers	....	....	....
(17) Longitude and time	....	....	....
(18) Square root	....	....	....
(19) Percentage generally	....	....	....
(20) Law of the right triangle	....	....	....
(21) Mensuration rules generally	....	....	....

(Please add any others.)

4. Should literal notation be taught (*a*) in the seventh grade? ....(*b*) in the eighth grade?...........

5. Should algebraic solution of equations be taught (*a*) in the seventh grade?...........(*b*) in the eighth grade?...........

6. Should literal problems (not equations) be presented (*a*) in connection with numerical problems of the same class? ....(*b*) separately, as a review of arithmetical principles?...........

7. Should problems in the evaluation of formulæ be given when the formulæ themselves cannot be derived by the students?...........

8. Should the idea of negative numbers be presented (*a*) in the seventh grade?.........(*b*) in the eighth grade?.........

9. Should the idea of approximate values of magnitudes and percentage of error be clearly defined (*a*) in the seventh grade? ....(*b*) in the eighth grade?..........

10. (Please answer the following as suggested in note (*e*) above.) Should the following topics be presented (assuming that the algebraic examples are to be extremely simple)? If so in which year?

	Numerically.	Literally.	In Equations.
Notation and numeration ...	....	....	....
Addition of integers	....	....	....
Subtraction do	....	....	....
Multiplication do	....	....	....
Division do	....	....	....
Decimals	....	....	....
U. S. Money	....	....	....
Reduction of fractions	....	....	....
Operation on fractions	....	....	....
Factoring	....	....	....
H. C. F.	....	....	....
L. C. M.	....	....	....
Average	....	....	....
Ratio and simple proportion .	....	....	....
Compound proportion	....	....	....
Partitive proportion	....	....	....
Partnership	....	....	....
Alligation	....	....	....
Specific gravity	....	....	....
Long measure	....	....	....
Square measure	....	....	....
Cubic measure and capacity .	....	....	....
Dry measure	....	....	....
Liquid measure	....	....	....
Avoirdupois weight	....	....	....

Troy and apothecary's wt.[1]	....	....	....
Metric system	....	....	....
English money	....	....	....
French money	....	....	....
German money	....	....	....
Circular measure	....	....	....
Longitude and time	....	....	....
Board measure	....	....	....
Plastering and painting	....	....	....
Temperature scales	....	....	....
Percentage	....	....	....
Profit and loss	....	....	....
Commission	....	....	....
Commercial discount	....	....	....
Insurance	....	....	....
Life insurance	....	....	....
Taxes	....	....	....
Stocks and bonds	....	....	....
Duties and customs	....	....	....
Simple interest	....	....	....
Exact interest	....	....	....
Bank discount	....	....	....
True discount	....	....	....
Domestic exchange	....	....	....
Foreign exchange	....	....	....
Equation of payments	....	....	....
Powers and exponents	....	....	....
Square root	....	....	....
Cube root	....	....	....
Mensuration of plane figures	....	....	....
Rectangles	....	....	....
Parallelograms	....	....	....
Triangles	....	....	....
Trapezoids	....	....	....
Circles	....	....	....
Regular polygons	....	....	....
Mensuration of solid figures.	....	....	....
Rectangular solids	....	....	....
Prisms	....	....	....
Pyramids	....	....	....
Cylinders	....	....	....
Cones	....	....	....
Spheres	....	....	....
Levers	....	....	....

[1] *Note.*—Many of the topics mentioned are palpably unfit for algebraic treatment. But an examination of the exercises in almost any text-book designed for first year high school work shows the inclusion of a surprisingly large number of topics.

Pulleys	....	....	....
Wheel and axle	....	....	....
Inclined plane	....	....	....
Similar plane figures	....	....	....
Similar solids	....	....	....
Uniform motion	....	....	....
Accelerated motion	....	....	....

11. Should the arrangement of the course be mainly topical (*a*) in the seventh grade?...........(*b*) in the eighth grade? ....

12. Should the plotting of graphs of statistics be taught (*a*) in the seventh grade?...........(*b*) in the eighth grade? ....

13. Should the plotting of graphs of simple algebraic functions be taught (*a*) in the seventh grade?...........(*b*) in the eighth grade?...........

14. Should simple exercises in concrete geometry with geometrical drawing be taught (*a*) in the seventh grade?.......... (*b*) in the eighth grade?...........

15. Should geometrical drawing be correlated (*a*) with drawing?...........(*b*) with mathematics?..........

16. Should mathematical exercises in the seventh and eighth grades be standardized? (*a*) according to relative difficutly of underlying concepts?...........(*b*) according to the number of operations entering the particular exercises?...........

Please add any further suggestions.